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Non- Linear Inverted Pendulum System by Multirate Output Feedback

Swami B.S.¹, Nagrale R.M.²

P.G. Student, Department of Control System Engineering, MBES Engineering College, Maharashtra, India¹

Associate Professor, Department of Control System Engineering, MBES Engineering College, Maharashtra, India²

ABSTRACT: In this paper Multirate output feedback (MROF) based control method for non linear systems is designed. In this method two controllers, (1) linear quadratic regulator and (2) fast output sampling, in which the system output is sampled at the faster rate as compared to control input. In the first approach of fast output sampling, the states of the system can be computed from the output of the system. While in second approach, previous control input along with past system outputs are used to calculate the present control input. These controllers are the part of MROF in which states of system can computed from output of the system.

KEYWORDS: Fast output sampling technique, Sampling feedback, Simulink.

I. INTRODUCTION

The aforementioned control strategies are based on full-state feedback. Using the concept multirate output feedback where in the system output is sampled at a faster rate compared to the control input. Consider the system sampled with a sampling time of $\Delta=T/N$ sec. Where N is an integer greater than the observability index of the system. Inverted pendulum is a difficult system to control, it is inherently unstable and non linear system. To control the pendulum it need to treated as linear, without error. Here, inverted pendulum can be control using three controllers, first one is LINEAR Quadratic Regulator, Second Is Fast Output Sampling with modified and new modified, those controllers have simulation result which will be compared to know the effect of different control technique Inverted pendulum is controlled by using the basic concept of space satellite, rockets, landing of aircrafts, balancing of ship against tide, seismometer. The system is non-linear, but for, if the operating region is small then, system is approximates to linear. The case where the system dynamics are described by a set of linear differential equation and the cost is described by a quadratic function is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR), a feedback controller whose equations are given below. The chapter presents a brief description of the LQR concept. Since, the choice of the LQR is the key towards design. The detailed analysis of the simulation and experimental results is presented. Fast output sampling is a kind of MROF in which the system output is sampled at a faster rate as compared to the control input. As constant gains are used in this technique practical implementation of this controller is easy. Instead of using the state observer the Fast Output Sampling controller can be used to realize the effect of state feedback gain by output feedback and then applied to the discretized system. The inverted pendulum is a classical control problem, which involves developing a system to balance a pendulum. For visualization purposes, this is similar to trying to balance a broomstick on a finger. There are three main subsystems that compose this design: (1) the mechanical system, (2) the feedback network which includes sensors and a method to read them, and (3) a controller and its interface to the mechanical system. After determining sets of requirements between the subsystems, each one can be designed independent of the other two, simplifying the design process.

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II. REALATED WORK

1. Linear Quadratic control

The chapter presents a brief description of the LQR concept.

Consider a control system -

$$\dot{x} = Ax + Bu \quad (1)$$

We shall choose the control signal to be

$$u = -k_x \quad (2)$$

Performance index is -

$$J = \int_0^{\infty} (x^T Qx + u^T Ru) dt \quad (3)$$

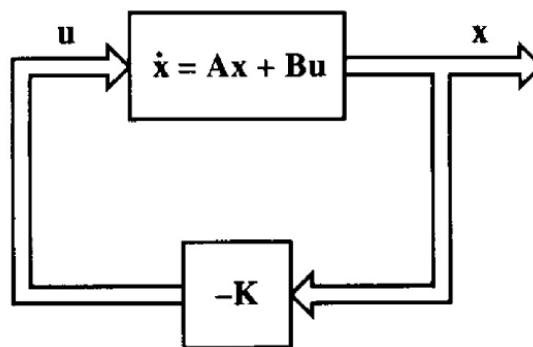


Fig. 1 optimal control system

To solve the optimizing problem Substituting (2) in (1), we get,

$$\dot{X} = (A - BK)X \quad (4)$$

The solution of this equation is given by,

$$X = e^{(A-BK)t} X(0) \quad (5)$$

If matrix K is chosen properly, the matrix A - BK can be made asymptotically stable matrix, and for all x(0) ≠ 0 it is possible to make x(t) approach zero as t approaches infinity.

Performance index J can be evaluated as

$$J = \int_0^{\infty} X^T (Q + K^T R K) X dt = -x^T P x |_0^{\infty} \quad (6)$$

$$J = -x^{\infty} p x^{\infty} + x^0 p x^0 \quad (7)$$

The minimization of J is obtained by solving the algebraic Riccati equation

$$\lambda(t) = P(t)X(t) \quad (8)$$

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State feedback gain is designed using matrix,

$$Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, R = 1$$

2. Fast Output Sampling :-

Fast output sampling is a kind of MROF in which the system output is sampled at a faster rate as compared to the control input.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (10)$$

$$y(t) = Cx(t) \quad (11)$$

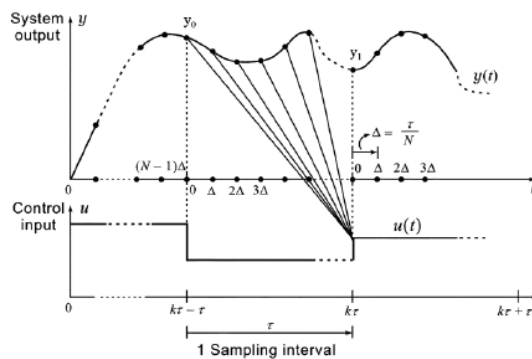


Fig. Visualization of fast output sampling feedback

As illustrate in Figure. When the system is sampled at the rate of $\frac{1}{\Delta}$ the equivalent discrete-time system is,

$$x(k+1) = \varphi x(k) + \tau u(k) \quad (11)$$

$$y(k) = C_x(k) \quad (12)$$

$$x(k+1) = \varphi_{\Delta} x(k) + \tau_{\Delta} u(k) \quad (13)$$

Where, $\Delta = \frac{1}{N}T$ In FOS, output measurements are taken at time instants $t = l\Delta$, where $l=0,1,2,\dots,N-1$. The control signal which is applied during the interval $KT \leq t \leq (K+1)T$, Where $l = 0,1,2,3,\dots,N-1$; by using these values in above equation we get some values which is illustrate in matrix form as follow –

$$y_k = \begin{bmatrix} y_0 \\ y_{\square} \\ y_{2\square} \\ \vdots \\ y_{(N-1)\square} \end{bmatrix}; C_0 = \begin{bmatrix} C \\ C\Phi_{\Delta} \\ C\Phi_{\Delta}^2 \\ \vdots \\ C\Phi_{\Delta}^{N-1} \end{bmatrix}; D_0 = \begin{bmatrix} 0 \\ C\Gamma_{\Delta} \\ C\Phi_{\Delta}\Gamma_{\Delta} + C\Gamma_{\Delta} \\ \vdots \\ C\sum_{j=0}^{N-2} \Phi_{\Delta}^j \Gamma_{\Delta} \end{bmatrix}$$

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From the above matrix the output equation is

$$y_k = C_0 x_k + D_0 u_k \quad (14)$$

Output feedback signal is given as,

$$u(t) = Ly_k \quad (15)$$

$$u_{(k)} = Fx_k \quad (16)$$

Sampling equation will be given as

$$y_{k+1} = (C_0 + D_0 F)(+ \tau F)^{-1} \quad (17)$$

$$u(t) = L\bar{C}x(k) \quad (18)$$

By comparing above equation

$$F = L\bar{C} \quad (19)$$

3. Modified approach of fast output sampling

Modified approach of fast output sampling technique

$$y_{(k+1)} = C_0 x_k + d_0 u_k \quad (20)$$

$$x(k) = L_y y_{(k)} + L_u u_{(k-1)}$$

Where,

$$L_u = \tau - \varphi(C_0^T C_0)^{-1} C_0^T D_0$$

$$L_y = \varphi(C_0^T C_0)^{-1} C_0^T$$

State variable is calculated by using $y(k)$ and $u_{(k-1)}$

Consider the equation as-

$$u(k) = F\varphi(C_0^T C_0)^{-1} C_0^T y_k + F[\tau - \varphi(C_0^T C_0)^{-1} C_0^T D_0] u_{(k-1)} \quad (21)$$

4. New modified approach for fast output sampling technique

By adding the past output $y_{(N-1)\Delta}$ to y_k

$$y_k = \begin{bmatrix} y_0 \\ y_\Delta \\ y_{2\Delta} \\ \vdots \\ y_{(N-1)\Delta} \end{bmatrix} = \begin{bmatrix} y_{(k-1)\tau} \\ y_{(k-1)\tau+\Delta} \\ y_{(k-1)\tau+2\Delta} \\ \vdots \\ y_{(k\tau-\Delta)} \end{bmatrix} \quad \bar{y}_k = \begin{bmatrix} y_{(k-1)\tau-\Delta} \\ y_{(k-1)\tau} \\ y_{(k-1)\tau+\Delta} \\ \vdots \\ y_{(k\tau-\Delta)} \end{bmatrix} = \begin{bmatrix} y_{-\Delta} \\ y_0 \\ y_\Delta \\ \vdots \\ y_{(N-\Delta)} \end{bmatrix} \quad \bar{y}_{(k+1)} = C \begin{bmatrix} x_{(k\tau-\Delta)} \\ x_{(k\tau)} \\ x_{(k\tau+\Delta)} \\ \vdots \\ x_{(k\tau+(N-1)\Delta)} \end{bmatrix}$$

But we have a equation,

$$x_{(k\tau)} = \varphi_\Delta x_{(k\tau-\Delta)} + \tau_\Delta u_{(k-1)\tau}$$

$$x_{(k\tau-\Delta)} = \varphi_\Delta^{-1} x_{k\tau} - \varphi_\Delta^{-1} \tau_\Delta u_{(k-1)\tau}$$

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From above equations,

$$\bar{y}_{(k+1)} = C \begin{bmatrix} \Phi_{\Delta}^{-1} x_{(k\tau)} - \Phi_{\Delta}^{-1} \Gamma_{\Delta} u_{(k-1)\tau} \\ x_{(k\tau)} \\ \Phi_{\Delta} x_{(k\tau)} + \Gamma_{\Delta} u_{(k\tau)} \\ \Phi_{\Delta}^2 x_{(k\tau)} + \Phi_{\Delta} \Gamma_{\Delta} u_{(k\tau)} \\ \vdots \\ \Phi_{\Delta}^{N-1} x_{(k\tau)} + \sum_{j=0}^{N-2} \Phi_{\Delta}^j \Gamma_{\Delta} u_{(k\tau)} \end{bmatrix}$$

From above matrix,

$$u_{(kT)} = F \Phi [(P_0^T P_0)^{-1} P_0^T] y_k + F ([0 \quad \Gamma] - \Phi (P_0^T P_0)^{-1} P_0^T S_0) \begin{bmatrix} u_{(k-2)T} \\ u_{(k-1)T} \end{bmatrix}$$

From the above equations we can use these on simulation to obtain best performance. Thus the gain of MROF is $K = [-0.3162 \quad -1.4440 \quad -56.5962 \quad -10.4046]$. After 0.07 second, the cart is forced 40N/m force to move the cart from the 0.1 of its initial condition. At the same time, the pendulum rod must remain in straight position. By the use of FOS on new modification, it is observed that the cart position have settle time 10.72 second and pendulum angle stable in 1.6 second as compared to the other controller. The settling time is reduced by 75%.

III. INVERTED PENDULUM SYSTEM

The inverted pendulum is a classical control problem, which involves developing a system to balance a pendulum. For visualization purposes, this is similar to trying to balance a broomstick on a finger. The mechanical design involved building a track, cart, pendulum, and drive mechanism. The inverted pendulum is a system that has a cart which is programmed to balance a pendulum as shown by a basic block diagram. An ideal controller would keep the pendulum balanced with very little change in the angle θ , or cart displacement, q .

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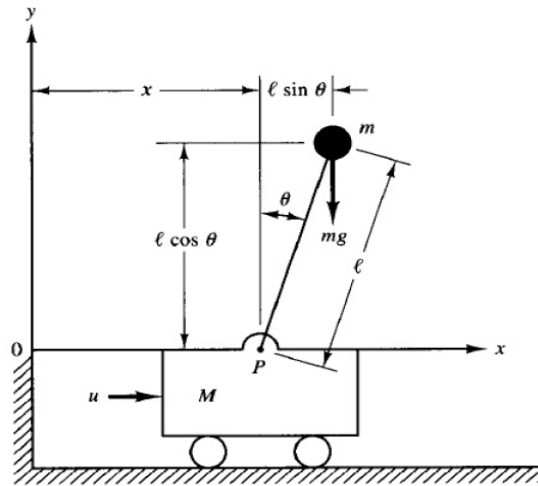


Fig. Inverted parametric presentation

Dynamics of inverted pendulum –

$$x_G = x + l \sin \theta$$

$$y_G = l \cos \theta$$

Horizontal motion of centre of gravity of inverted pendulum rod is given by,

$$m \frac{d^2 x_G}{dt^2} = m \frac{d^2 (x + l \sin \theta)}{dt^2} = H$$

Similarly, vertical motion of centre of gravity of inverted pendulum rod is given by,

$$m \frac{d^2 y_G}{dt^2} = m \frac{d^2 (l \cos \theta)}{dt^2} = V - mg$$

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t)$$

Where, A is a system matrix, B is a input matrix, and C is a output matrix.

To obtain the value of A,B,C we have to consider equation,

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta} = u$$

$$(J + ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$$

Table-1 gives the information about symbols and prescription used in the work.

M	Mass of motor driven cart	(2.4),Kg
m	Mass of pendulum	(0.23),Kg
b	Friction of cart	(0.0),N/m/s
J	Moment of inertia of pendulum	(0.0),Kg ²
l	Length of pendulum	(0.36),m
g	Acceleration due to gravity	(9.81),m/s ²
u	Force applied to cart	N
x	Cart position	m
?	Pendulum angle	rad

Table1:-Inverted pendulum system parameter.

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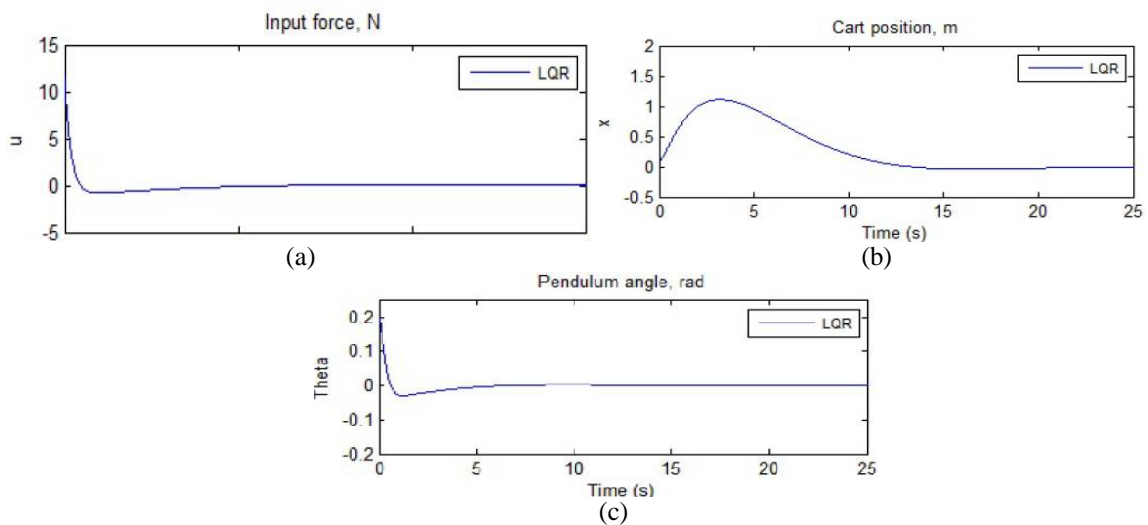
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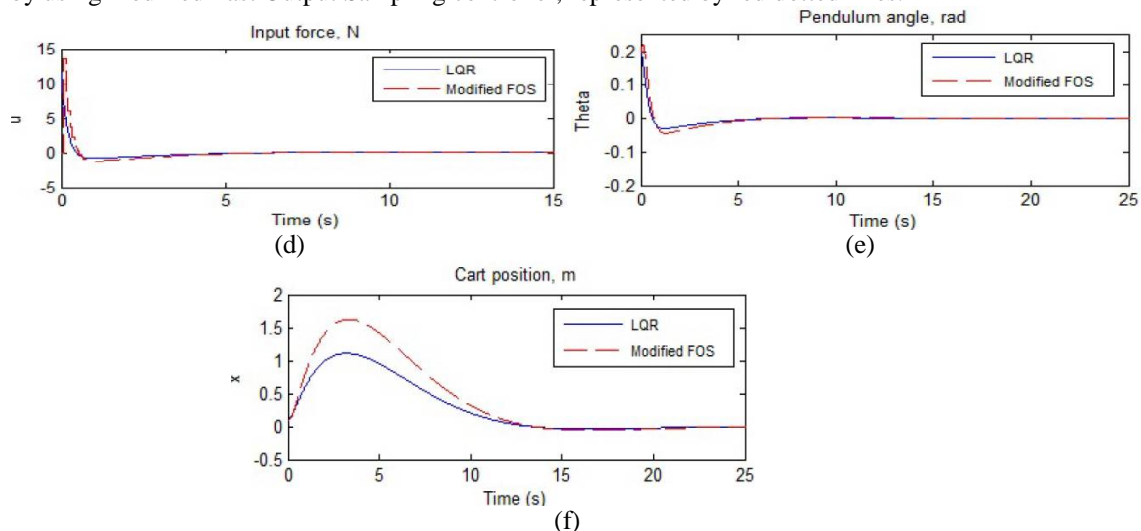
IV. EXPERIMENTAL RESULTS

Figures shows the results of simulink , waveforms are seprated with the different controllers used.

- (1) Result (a),(b),(c) are generated using LQR controller, basically in this given input force shown by (a),effect of cart position is shown by (b), and also fig.(c) shows the effect on pendulum angle represented by blue colored line.



- (2) Figure (d),(e),(f) shows the result about inout force,pendulum angle,cart position respectively, which are obtained by using modified Fast Output Sampling controller, represented by red dotted lines.



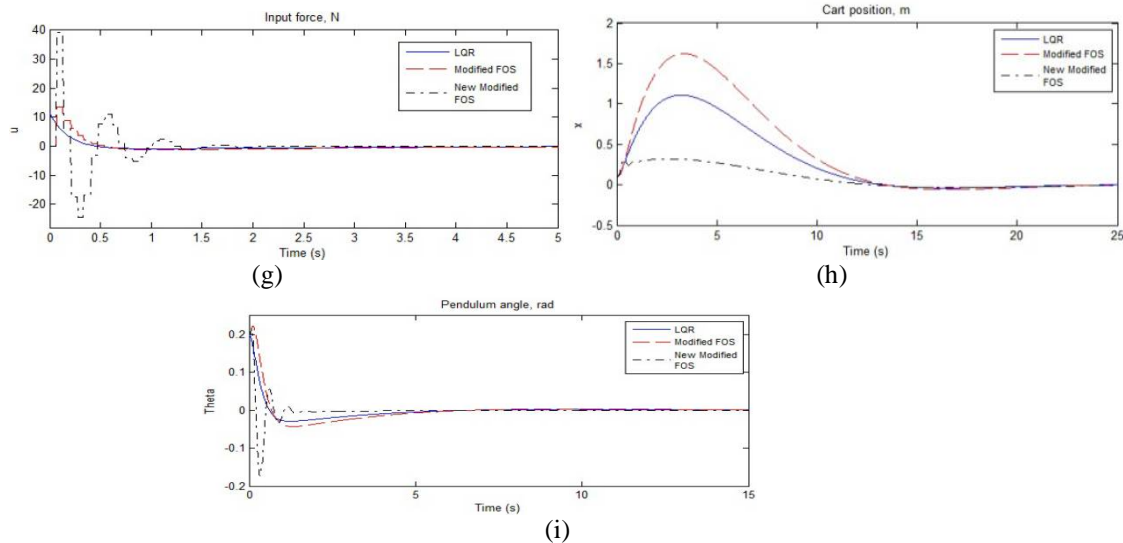
- (3) In the sense of required a better result about controlling the pendulum in minimum time another modification is done on Fast output controller. Figure (g),(h),(i) shows the results from new modified Fast Output Sampling with input force, cart position, and pendulum angle respectively, represented by small dotted lines.

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V. CONCLUSION

An inverted pendulum is stabilized in less time by sampling the control input in delay. Output is compared with the state feedback controller and modified FOS technique to show the efficient method. We can find an optimal solution by multirate output feedback method.

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